Autotuning for Model-Based PID Controllers

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The autotuning test of Åström and Hägglund (1984) is conducted by using a relay in the feedback loop to produce periodic cycles so that the ultimate gain and ultimate frequency of the process can be derived. In the autotuning (AT) procedure of Åström and Hägglund, such ultimate data are used to find the PID controller settings based on Z-N rules (Ziegler and Nichols, 1942). Although it was found that such data are always accompanied with errors (sometimes as high as 20%), the effect of such errors to the performance of PID loops would be immaterial.

However, as the Z-N method is aimed at one-quarter decay ratio, the resulting control loop usually has excessive oscillations that lead to poor performance. To achieve better performance, many other tuning methods which use model-based designs such as IMC (Rivera et al., 1986) or integral performance criteria such as ISE, IAE, ITAE, etc. (Lopez et al., 1967) have to be considered. However, to apply these tuning rules, there needs to be a proper parametric model such as the FOPDT. This brought forth quite a few studies (Luyben et al., 1991; Li et al., 1991) that aim at identifying such models by using the AT test. But, to apply such models derived from the conventional AT test, one should consider the following facts: if the process is truly of FOPDT, the frequency data will be accompanied with errors which would lead to an erroneous ratio of θ/τ; on the other hand, if the process is of higher order, the FOPDT model derived from the AT test would have a lower value of θ/τ than other parametric fitted models. Since θ/τ ratio of a FOPDT model is essential for tuning a model-based PID controller, the controller thus obtained would be tightly tuned and the control system would be apt to have more oscillatory responses when subjected to modeling errors. It has been noticed (Smith, 1972) that a FOPDT model that has a better fit to the step response of the true process would have a larger value of θ/τ that would lead to a more robust tuning and a minimal ITAE design. It is thus the purpose of this article to propose an autotuning method using a proposed autotuning test. The proposed AT test is similar to the one of Åström and Hägglund except the output is tailed-off at the end. The amplitude and the period of the constant cycles together with input and output moments in one run are used to compute the parameters of the FOPDT model. The resulting FOPDT model is then used to determine a model-based PID controller.

When applying this proposed method to a true FOPDT process, the resulting FOPDT model for autotuning would have perfect match. On the other hand, as an approximation to higher-order processes, the derived FOPDT model for autotuning would emphasize having a good fit to the step response of the true process. Such an FOPDT model usually has a close ultimate frequency but a smaller ultimate gain compared with those from the conventional AT test. A smaller ultimate gain would lead to a smaller loop gain when applying those existing tuning formula for PID controllers. Consequently, the resulting model-based controller would not endanger the stability but still can have satisfactory and robust control performance.

Proposed Autotuning Testing

Åström and Hägglund (1984) used a relay feedback to replace a conventional feedback controller to perform the autotuning test. The relay has $+h$ and $-h$ at the output. The system responds to this bang-bang relay output with limit cycles. The resulting period P and amplitude a of the cycles under symmetric relay are then used to calculate the ultimate gain k_u and the ultimate frequency w_u according to results from the describing function

$$\omega_u = \frac{2\pi}{P} \tag{1}$$

$$k_u = \frac{4h}{a\pi} \tag{2}$$

The proposed autotuning test follows mostly the conventional AT test with only slight modification. The testing is carried out in the following manner:

(1) The magnitude at the relay output is changed to $+\gamma h$ and $-h$, i.e., the output can be symmetric ($\gamma = 1$) or unsymmetrical ($\gamma \neq 1$).

(2) When the output has constant oscillation for a few (usually three) cycles, the relay is turned off, i.e., $u = 0$.

(3) The test is complete when the process output becomes almost constant.

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The proposed test differs mainly from the conventional one only in steps 2 and 3. Because of step 2, the resulting response from this test would tail off because the process input is restored to its original steady-state value.

In the following texts, we shall designate the positive peak height at constant cycling as a_1 and negative peak height as a_2 ($a_2 < 0$), and the period of cycling as P, respectively. The average distance from the positive peak to the negative peak is also designated as a, i.e.,

$$a = \frac{a_1 - a_2}{2}$$

When $\gamma = 1$, the relay is symmetric so we would have almost the same autotuning response as that of Åström and Hägglund except the tail part. Therefore, ultimate data can be read from this response, too. On the other hand, when $\gamma \neq 1$, the relay will be unsymmetrical.

Estimation of FOPDT Models for Autotuning

In general, chemical processes are of high order. As a reality, it is common to use the FOPDT model as an approximation to such processes for control practice. In the following, we shall depict how to estimate a FOPDT model based on the results of the proposed AT test.

Let y_f, a, and P denote the final offset, the amplitude, and the cycling period obtained from the proposed AT test, respectively. The parameters of a FOPDT model are estimated as follows

(1) Equivalent Dead Time, $\hat{\theta}$

$$\hat{\theta} = \frac{P}{2 - \dfrac{A+B}{\ln\left(1 - \dfrac{2}{\gamma+1}\dfrac{a}{k_p h}\right)}} \tag{3}$$

where

$$A = \ln\left[\frac{k_p h - y_f + (\gamma k_p h + y_f)\left(\dfrac{2}{\gamma+1}\dfrac{a}{k_p h}\right)}{k_p h - y_f}\right]$$

$$B = \ln\left[\frac{\gamma k_p h + y_f + (k_p h - y_f)\left(\dfrac{2}{\gamma+1}\dfrac{a}{k_p h}\right)}{\gamma k_p h + y_f}\right]$$

(2) Process Steady-State Gain, k_p

The estimation of the process steady-state gain is based on the zero-order moments of the input and output of one cycle when the system becomes to oscillate with constant cycles. That is

$$\hat{k}_p = \frac{\int_{t_0}^{t_0+P}[y(t) - y_f]dt}{\int_{t_0}^{t_0+P}u(t)dt} = \frac{\int_{t_0}^{t_0+P}y(t)dt - y_f P}{\int_{t_0}^{t_0+P}u(t)dt} \tag{4}$$

where t_0 is any time origin for integration.

(3) Process Time Constant, τ

The time constant is estimated indirectly by computing α which has the following well known relation

$$\alpha = \int_0^\infty [1 - S(t)]dt = \frac{-\left[\dfrac{dG_p(s)}{ds}\right]_{s\to 0}}{k_p} \tag{5}$$

where $S(t)$ is the *unit step response* of the process. For a FOPDT model, α becomes

$$\alpha = \tau + \theta \tag{6}$$

The parameter α is calculated according to whether the process output at t_f y_f is zero or not by the following

when $y_f = 0$

$$\alpha = \frac{\int_0^\infty t[y(t) - k_p u(t)]dt}{k_p\int_0^\infty u(t)\,dt} \tag{7}$$

when $y_f \neq 0$

$$\alpha \approx \frac{k_p\int_0^{t_f}u(t)dt - \int_0^{t_f}y(t)dt + y_f t_f}{y_f} \tag{8}$$

where t_f is the final time of the AT test.

Once α and the equivalent dead time $\hat{\theta}$ are calculated, the time constant can be estimated according to Eq. 6

$$\hat{\tau} = \alpha - \hat{\theta} \tag{9}$$

(4) Remarks

There are a few things to be remarked concerning the above estimations.

(a) Equation 3 is derived from a true FOPDT process which responds to a relay feedback controller according to the following

$$y(t_0 + t) = y(t_0)e^{-t/\tau} + \frac{k_p}{\tau}\int_{t_0}^{t_0+t} e^{-(t_0+t-\xi)/\tau}u(\xi - \theta)d\xi$$

(b) It can be shown that, for a true FOPDT process, the ratio of P/θ from a conventional AT test would become

$$\frac{P}{\theta}=2\left[1+\frac{\ln(2-e^{-\theta/\tau})}{\theta/\tau}\right] \quad (10)$$

compared to the theoretical value of P_u/θ which follows

$$\tan^{-1}\left[2\pi\left(\frac{P_u}{\theta}\right)^{-1}\left(\frac{\theta}{\tau}\right)^{-1}\right]+2\pi\left(\frac{P_u}{\theta}\right)^{-1}=\pi \quad (11)$$

On the other hand, the value of $\hat{k}_u k_p$ will be

$$\hat{k}_u k_p=\frac{4hk_p}{\pi a}=\frac{4}{\pi(1-e^{-\theta/\tau})} \quad (12)$$

in comparison with the theoretical value obtained from the amplitude condition at critical frequency

$$k_u k_p=\sqrt{4\pi^2\left(\frac{P_u}{\theta}\right)^{-2}\left(\frac{\theta}{\tau}\right)^{-2}+1} \quad (13)$$

The results in Eqs. 10, 11, 12, and 13 are plotted in Figure 1. From this figure, it is then clear that those estimated ultimate data obtained from the conventional AT approach would always accompany with errors. This error is part of nature of the conventional AT test and cannot be removed no matter how carefully the experiment is conducted or how carefully the data are collected. However, if the process is truly of FOPDT, then by the estimation procedures given above, the resulting model would have a perfect match to the real process.

(c) Use of α in the estimation lies on the fact that it stands for the area between its final value and dynamic response curve. Thus, to match the value of α from a model means that the model has almost the same dynamic behavior as the process during the transient period when it responds to a step input.

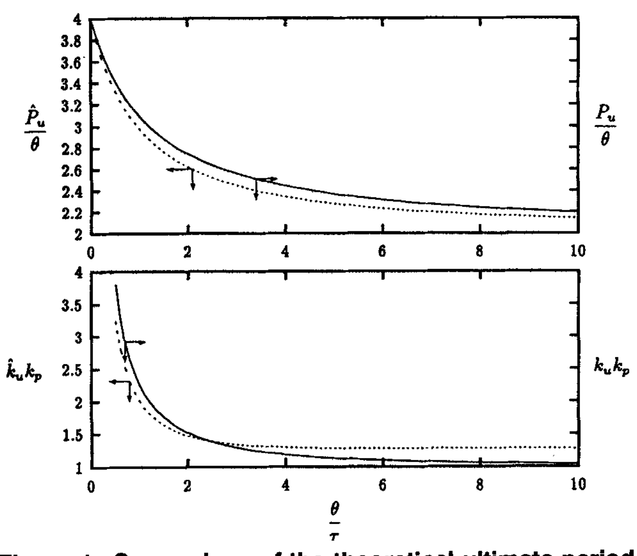

Figure 1. Comparison of the theoretical ultimate period and ultimate gain with those estimated from the conventional autotuning test for FOPDT processes.

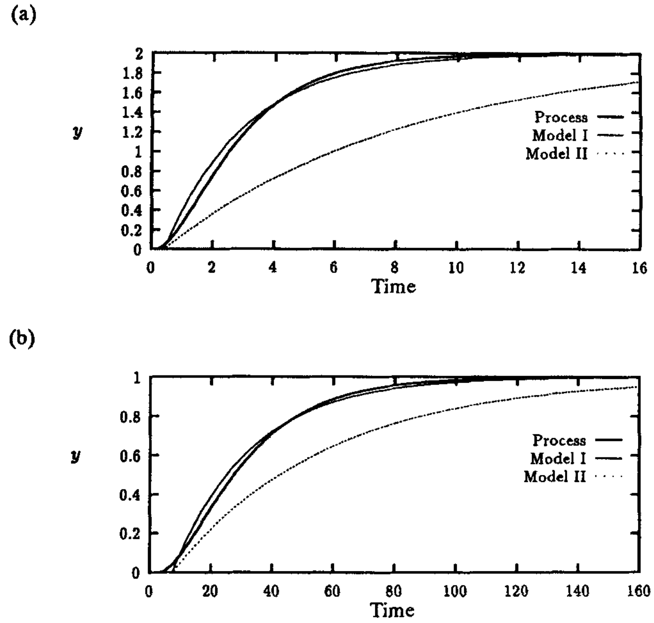

Figure 2. Step responses: Model I from autotuning test vs. Model II from the exact ultimate frequency and ultimate gain.

(a) $G_1(s)=\frac{2e^{-0.1s}}{(2s+1)(s+1)}$;

(b) $G_2(s)=\frac{e^{-2s}}{(20s+1)(10s+1)(s+1)}$.

(d) The existence of y_f means that after the process input is reset to its original steady-state value, the process output does not return to its original steady-state value. There are two possible situations when this phenomena may occur. One possibility originates from the starting point of test being not a real steady state. The other possibility is the occurrence of a constant unknown disturbance which enters the process at the same point as u. Because of this unknown bias at the input, both amplitude and period of the test will change.

Two example processes are used for demonstration. They are

$$\text{Process 1} \quad G_1(s)=\frac{2e^{-0.1s}}{(2s+1)(s+1)}$$

$$\text{Process 2} \quad G_2(s)=\frac{e^{-2s}}{(20s+1)(10s+1)(s+1)}$$

The step responses of these two processes and their models derived from using the proposed AT test ($\gamma=2$) are shown in Figure 2. The results of estimations are summarized in the following:

- For Process 1, we have: $P=1.81$, $a_1=0.176$, $a_2=-0.0904$, and $\alpha=3.09$. The resulting model [designated as Model I, $\overline{G}_1^I(s)$] becomes

$$\overline{G}_1^I(s)=\frac{2e^{-0.42s}}{2.67s+1}$$

If the model that is derived from the exact ultimate data (i.e., $\omega_u = 3.841$ and $k_u = 15.38$) is designated as model II, then we have

$$\overline{G}_1^{II}(s) = \frac{2e^{-0.42s}}{8.00s+1}$$

• For Process 2, we have: $P = 30.81$, $a_1 = 0.238$, $a_2 = -0.123$, and $\alpha = 32.98$. The resulting model [designated as Model I, $\overline{G}_2^I(s)$] becomes

$$\overline{G}_2^I(s) = \frac{e^{-7.45s}}{25.53s+1}$$

The model that is derived from the exact ultimate data (i.e., $\omega_u = 0.2189$ and $k_u = 11.06$) is

$$\overline{G}_2^{II}(s) = \frac{e^{-7.59s}}{50.33s+1}$$

It should be noticed that, in both cases, if an unknown biased input $d = 0.05$ is introduced during the tests, there are only insignificant changes on the parameters.

Model-Based PID Tuning

The model-based PID controllers refer to those controllers whose PID settings are derived from using a parametric transfer function model, especially the FOPDT one. One such category is the controller that uses integral performance criteria such as ISE, ITSE, IAE, and ITAE (Lopez et al., 1967). The PID controller considered is of the following

$$G_c(s) = k_c\left(1 + \frac{1}{\tau_I s} + \frac{\tau_D s}{\beta \tau_D s + 1}\right) \qquad (14)$$

where β is a constant taken as 0.1 for the simulation study. Tuning rules for the PID parameters are given in the form of

$$z = C\left[\frac{\theta}{\tau}\right]^D \quad \text{or} \quad z = C + D\left[\frac{\theta}{\tau}\right] \qquad (15)$$

where $z \equiv k_c k_p$ for proportional mode, τ/τ_I for integral mode, and τ_D/τ for derivative mode. The constants C and D are tabulated for either setpoint changes or load changes and can be found elsewhere (Smith and Corripio, 1985).

Another popular model-based design for PID controllers is known as IMC-PID (Rivera et al., 1986). The PID controller considered is of the following (Morari and Zafiriou, 1989)

$$G_c(s) = \frac{k_c\left(1 + \dfrac{1}{\tau_I s} + \tau_D s\right)}{\tau_F s + 1} \qquad (16)$$

Parameters for the above PID controller are

$$k_c = \frac{1}{\hat{k}_p}\frac{\hat{\tau} + \hat{\theta}/2}{\epsilon + \hat{\theta}} \qquad \tau_I = \hat{\tau} + \frac{1}{2}\hat{\theta}$$

$$\tau_D = \frac{\hat{\tau}\hat{\theta}}{2\hat{\tau} + \hat{\theta}} \qquad \tau_F = \frac{1}{2}\frac{\epsilon\hat{\theta}}{\epsilon + \hat{\theta}} \qquad (17)$$

where ϵ denotes the adjustable parameter for the IMC controller.

From the tuning formula given above, it is noticed that the proportional gain in each case is approximately proportional to the inverse of θ/τ. Thus the ratio of θ to τ is essential to these model-based controllers, especially when the ratio is small. Ho et al. (1995) investigated the gain and phase margins of such model-based PI controllers and found that such PI controllers have their gain margins around three. However, from the results of the previous examples, τ/θ for those derived from ultimate data of the conventional AT test may be as high as three times of that from the proposed AT test. As a result, the PID controllers derived from using ultimate data would have more oscillations and larger overshoots. On the other hand, the Z-N tuned PID controller has lower gain margin when θ/τ is small. Consequently, it is apt to overtune the system and would produce more persistent oscillations unless some precautions are taken.

An example of servo tracking control for $G_1(s) = [2e^{-0.1s}/(2s+1)(s+1)]$ is used to illustrate the autotuning that uses the proposed AT test. All the controller settings are given below

	k_c	τ_I	τ_D	τ_F
Z-N	9.047	0.818	0.204	···
ITAE(1)	2.346	3.437	0.148	···
ITAE(2)	5.995	10.097	0.159	···
IMC(1)	2.364	2.880	0.195	0.0652

where the adjustable parameter ϵ is chosen as $0.45\hat{\theta}$. Setpoint responses shown in Figure 3 are compared with those that use FOPDT models derived from those that use exact ultimate data. From Figure 3a, it is obvious to see that the Z-N controller results in excessive oscillations and thus has a larger value of ITAE (ITAE = 4.7185) than others. On the other hand, the ITAE or IMC controllers using the derived FOPDT model give almost the same responses which have smaller overshoots and less oscillations as expected. Although the ITAE (or the IMC) controllers using the FOPDT model derived from the exact ultimate data give quicker but more oscillatory responses, generally, this system is more sensitive to modeling errors, as shown in Figure 3b.

Conclusion

In this article, a modified autotuning test for deriving a FOPDT model for a model-based PID controller tuning is proposed. The estimations utilize the data from one single run of experiment and the resulting FOPDT model emphasizes a good fit to the step response of the true process. The FOPDT model derived in this manner as an approximation to higher-order systems usually has a close ultimate frequency but smaller ultimate gain compared to those from the

(a)

(b)

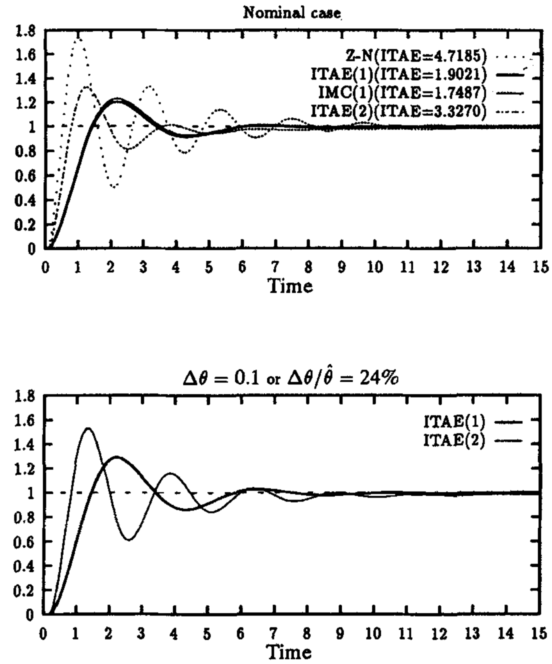

Figure 3. Comparison of setpoint responses for $G_1(s) = [2e^{-0.1s}/(2s+1)(s+1)]$.

(a) Nominal case: Z-N based on the exact ultimate frequency and ultimate gain, ITAE (1) and IMC (1) based on Model I, and ITAE (2) based on Model II; (b) Modeling error case: $\Delta\theta = 0.1$ or $\Delta\theta/\hat{\theta} = 24\%$.

conventional AT experiments. As a result, the derived model-based controllers (such as ITAE, IMC, etc.) would be more robust to modeling errors but still can have satisfactory performance. In contrast, such controllers if derived from ultimate data would be more sensitive to modeling errors. With the proposed autotuning test and the use of existing model-based tuning rules, an autotuning system for a PID controller can be easily formulated.

Notation

A, B = parameters in Eq. 3
$G_c(s)$ = controller transfer function
$G(s)$ = process transfer function
k_c = controller gain
P_u = ultimate period of the process
t_f = final time of the autotuning test
$u(t)$ = process input
$y(t)$ = process output
$\alpha = -[dG(s)/ds]_{s\to 0}/k_p$ or as defined in Eq. 5
γ = ratio of positive peak to negative peak of relay output
τ_F = time constant of the IMC filter

Superscripts

$\bar{}$ = model of the process
$\hat{}$ = estimated value

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